

Mathematics

- 7.10 The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real –life. The student is expected to
- (B) find the probability of independent events.
- 7.11 The student understands that the way a set of data is displayed influences its interpretation.
- (A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, Venn diagrams, and justify the selection.
 - (B) make inferences and convincing arguments based on analysis of given or collected data.

Technology Applications

The student demonstrates knowledge and appropriate use of hardware components, software programs, and their connections. The student is expected to

- (1)(B) compare, contrast, and appropriately use the various input, processing, output, and primary/secondary storage devices.
- (1)(C) demonstrate the ability to select and use software for a defined task according to quality, appropriateness, effectiveness, and efficiency.
- (1)(E) use technology terminology appropriate to the task.
- (1)(F) perform basic software application functions including, but not limited to, opening an application program and creating, modifying, printing, and saving documents.

The student uses data input skills appropriate to the task. The student is expected to

- (2)(A) demonstrate proficiency in the use of a variety of input devices such as mouse/track pad, keyboard, microphone, digital camera, printer, scanner, disk/disc, modem, CD-ROM, or joystick.

The student acquires electronic information in a variety of formats, with appropriate supervision. The student is expected to

- (5)(A) identify, create, and use files in various formats such as text, bitmapped/vector graphics, image, video, and audio files.

The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to

- (7)(G) integrate two or more productivity tools into a document including, but not limited to, tables, charts and graphs, graphics from paint or draw programs, and mail merge.

Materials

Advanced Preparation:

- Sign up for the computer lab.
- Have the **The Teacher Helper** file ready for students to access in the computer lab.
- Make the **Teacher Helper** transparency.

For each student:

- **The Helper Dilemma** activity sheet
- **The Choir Helper** activity sheet
- **Simulation** activity sheet
- Sticky notes

For whole class demonstration:

Transparency of **Teacher Helper**

For each student group of students:

- Coin
- 10-sided number decahedron
- Assortment of spinners, polyhedral-dice, marbles, or cards for **The Choir Helper** activity or materials for students to create their own

ENGAGE

The Engage portion of the lesson is designed to create student interest in the concepts addressed. Technology is not used in this phase since the focus of this activity is to remind students of sample spaces and probability. This part of the lesson is designed for whole group instruction and groups of 2 students.

1. Display the **Teacher Helper** transparency on the overhead.
2. Read the problem as a class and ask students to take a moment to think about the problem on their own. Have students record their solutions on a sticky note.
3. Prompt the students to work with a partner to share and/or compile their thoughts and answer the questions.
4. Debrief using the facilitation questions.
5. Extend the discussion to find other probabilities such as the probability of getting an even number, the number 11, etc.

Facilitation Questions – Engage Phase

- How many students are in Mrs. Alexander's class?
20
- How do you know?
Answers may vary. This will hopefully lead into a discussion on sample spaces.
- What is a sample space?
A sample space is the set of all possible outcomes for a given scenario.
- What is the sample space for this scenario?

<i>Heads, 1</i>	<i>Heads, 6</i>	<i>Tails, 1</i>	<i>Tails, 6</i>
<i>Heads, 2</i>	<i>Heads, 7</i>	<i>Tails, 2</i>	<i>Tails, 7</i>
<i>Heads, 3</i>	<i>Heads, 8</i>	<i>Tails, 3</i>	<i>Tails, 8</i>
<i>Heads, 4</i>	<i>Heads, 9</i>	<i>Tails, 4</i>	<i>Tails, 9</i>
<i>Heads, 5</i>	<i>Heads, 10</i>	<i>Tails, 5</i>	<i>Tails, 10</i>
- Are all of the possibilities equally likely? Why?
Yes, there is only one head and one tail. Also, each number occurs only one time.
- What is the probability of the student assigned to Head, 6 being the helper?
Have students refer back to the sample space. There is a $\frac{1}{20}$ chance for the student with a Head, 6 to be the helper. Connect the sample space to finding the independent probability.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of 2 students.

1. Distribute a 10-sided number decahedron and a coin to each pair of students.
2. Distribute **The Helper Dilemma** activity sheet to each student.
3. The students should perform the experiment and record results.
4. Take students into the computer lab to complete the activity.
5. Use the facilitation questions when students need help.

Facilitation Questions – Explore Phase

- How do you highlight a range of cells?
Click the mouse and hold inside the first cell and drag to the last cell needed. Let go of the mouse.
- How do you format cells?
Use the mouse and right click or choose "Format" in the menu bar and cells from the pull down menu.
- How do you start any formula in a spreadsheet document?
All formulas start with an equal sign.
- When inputting formulas is it better to use numerical values or cell locations? Why?
Cell locations are better since the value will automatically change in the formula cell if any numerical values are changed in the linked cells. However, numerical values are also appropriate at times as seen in the spreadsheet.
- What do you know about the problem?
Answers may vary. Have students verbalize the parts of the problem they know.
- What do you need to know?
Answers may vary. Have students verbalize the parts of the problem they need to know through questioning.
- Have you worked problems like this before? Explain.
Answers may vary. Relate the problem to prior learning and prior experiences.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief **The Helper Dilemma**.
2. Discuss notation for probability of compound events (i.e. P(head, one)).

Facilitation Questions – Explain Phase

- What is the difference between theoretical and experimental probability?
Answers may vary. Take this opportunity to review these topics.
- How were the experimental and theoretical probabilities the same numerically?
Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.

Facilitation Questions – Explain Phase

- How were the experimental and theoretical probabilities the same numerically?
Answers may vary. Depending on the experiment, some may say that the experimental probabilities were equally distributed like the theoretical probabilities.
- How were the experimental and theoretical probabilities different?
Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is helper so that when repeats occur she knows to flip the coin and roll the number decahedron again.
- If the fractions were changed to percents, what would you expect the percents to total and why?
Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.
- How could we use the spreadsheet to change the fractions to percents?
To change fractions to percents, highlight the desired cells and choose format cells.
- How could we use the spreadsheet to total the percent values?
To find a total, insert a formula by typing “=SUM” and highlighting or typing in the desired cells.
- If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
Answers may vary. Students may suggest that she use a numbered cube with 12 sides. Some students may suggest that she flip a coin, roll the numbered cube and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.
- What can you conclude about the class where Mrs. Alexander assigned tails to girls and heads and prime numbers to boys?
Answers may vary. Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.
- How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations?
Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.

Facilitation Questions – Explain Phase

- In a previous question you were asked, “If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?” If 3 items were used to find the helper, for example, a coin, a number cube and a spinner, how would the results be affected?
Answers may vary. Discuss that a third item would increase outcomes.
- How could we use the spreadsheet to help us record and calculate the results?
Answers may vary. To calculate theoretical probability, three columns will be needed on the spreadsheet. In the 4th column a formula will be inserted to multiply the probability of 3 events.
- Do the items always have to yield equally likely results?
Answers may vary. Discuss with students that they do not. Have students give examples where the outcomes are not equally likely.
- How does technology assist us in communicating our results?
Answers may vary. Some students may suggest that the graph is easier to interpret and helps by eliminating the fractions. Others may suggest that the ease in changing from fractions to percents helps in communicating results.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. This part of the lesson is designed for individual investigation.

1. Distribute **The Choir Helper** to each student.
2. Prompt students to complete #1-2 before going to the lab.
3. Prompt students to open the file **The Teacher Helper** spreadsheet and click on the tab titled **The Choir Helper**.
4. Students should complete the remainder of the worksheet using the spreadsheet.
5. Students should print the document when finished.
6. Ask Facilitation Questions as needed.

Facilitation Questions – Elaborate Phase

- How do you write a formula in a spreadsheet document?
Formulas start with the = sign.
- How do spreadsheets help you?
Answers may vary. Spreadsheets will do calculations needed with a formula.
- How is this activity similar and different to **The Helper Dilemma**?
Answers may vary. Still has two simulation items, but larger sample space.

Facilitation Questions – Elaborate Phase

- How do you make predictions from results?
Include a discussion here of scale factors. The simulation was for 50 trials, so to predict results for 100 trials use a scale factor of 2 (multiply by 2).
- How could you use a graph to show that the results for 50 trials, 100 trials, 250 trials, etc. are proportional?
Answers may vary. Discuss with students that a graph could be made charting each individual outcome. For example, chart the results for P(tail, letter A) for 50 trials by letting the x-axis represent trials and the y-axis represent outcomes. Other outcomes for the same probability could be graphed, and a discussion of the data points should follow. The points should appear to be in a straight line that would travel through the origin. Thus, the data is proportional.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute **Simulation** activity sheet to each student.
2. Upon completion of **Simulation** activity sheet, the teacher should use a rubric to assess student understanding of the concepts addresses in this lesson.

Answers and Error Analysis for selected response questions:

Question Number	TEKS	Correct Answer	Conceptual Error	Conceptual Error	Procedural Error	Procedural Error	Guess
1	7.10B	D	A		C		B
2	7.11B	B	A	D			C
3	7.11A	A	B	C			D
4	7.10B	A	D		B	C	

The Helper Dilemma – (Possible Answers)

1. Use a coin and a 10-sided number decahedron to simulate the experiment 40 times. Record your results in the frequency table.

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

2. Transfer your information into **The Teacher Helper** document. Follow the instructions in the orange boxes numbered 1-6.
3. Create a graph to represent the Theoretical Probability in Column B.
 - Highlight the Combinations (i.e. Head, 1) in Column A along with the data in the green cells in Column B.
 - Go to Insert Chart.
 - Choose "doughnut" for the chart type on the left-hand side.
 - Click on next twice and type in the title "Theoretical Probability."
 - Click on the tab that reads "Legend." Click in the box next to "Show Legend" so that the check mark disappears.
 - Click on the tab that reads "Data Labels." Click inside the boxes next to "Category Name and Value" so that a check mark appears in both boxes.
 - Click on finish.
 - Click and hold inside the chart. Drag the chart below the first set of data.
 - Enlarge the chart by clicking on a corner and dragging to the desired size.

(continue: The Helper Dilemma)

4. Create a graph to represent the Experimental Probability in Column I (include the Combinations such as Head, 1). Follow the same instructions as #3 except highlight the information in Columns H and I and use the title "Experimental Probability." Drag the chart next to the Theoretical Probability Chart, the first chart.
5. Print the document. Be sure to preview the pages to be printed. You may need to adjust margins so that you only print 1 or 2 pages.
6. How were the experimental and theoretical probabilities the same? Explain.
Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.
7. How were the experimental and theoretical probabilities different? Explain.
Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is the helper so that when repeats occur, she knows to flip the coin and roll the number decahedron again.
8. If the fractions were changed to percents, what would you expect the percents to total and why?
Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.
9. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
Answers may vary. Students may suggest that she use a number dodecahedron with 12 sides. Some students may suggest that she flip a coin, roll the number decahedron and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

10. What is the sample space for this class?

<i>Head, 1</i>	<i>Head, 6</i>	<i>Tail, 1</i>	<i>Tail, 6</i>
<i>Head, 2</i>	<i>Head, 7</i>	<i>Tail, 2</i>	<i>Tail, 7</i>
<i>Head, 3</i>	<i>Head, 8</i>	<i>Tail, 3</i>	<i>Tail, 8</i>
<i>Head, 4</i>	<i>Head, 9</i>	<i>Tail, 4</i>	<i>Tail, 9</i>
<i>Head, 5</i>	<i>Head, 10</i>	<i>Tail, 5</i>	<i>Tail, 10</i>

(continue: The Helper Dilemma)

11. What can you conclude about this particular class? Explain.

Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.

12. Which gender is most likely to be the helper? Explain.

A girl is most likely to be the helper since more combinations are assigned to girls than boys.

13. What is the probability of a girl being the helper? Explain.

There is a $\frac{1}{2}$ chance of getting a tail and a $\frac{10}{10}$ chance of getting a number on the decahedron. Combine the probabilities using multiplication, $\frac{1}{2} \cdot \frac{10}{10}$, to get a $\frac{10}{20} = \frac{1}{2}$ chance of getting a girl helper.

14. What is the probability of a boy being the helper? Explain.

There is a $\frac{1}{2}$ chance of getting a head and a $\frac{4}{10}$ chance of getting a prime number on the number decahedron. Combine the probabilities using multiplication, $\frac{1}{2} \cdot \frac{4}{10}$, to get a $\frac{4}{20} = \frac{1}{5}$ chance of getting a boy helper.

15. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.

The Choir Helper – (Possible Answers)

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher’s lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same “code”, Mr. Roberts can not use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

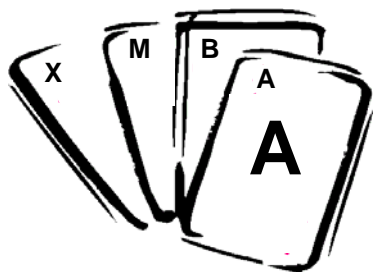


A Coin

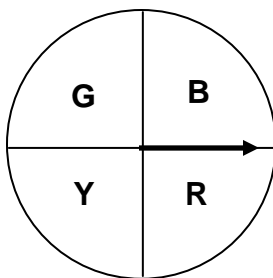


A Six-Sided Number Cube

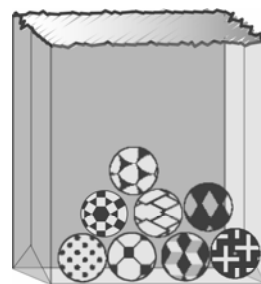
A 10-sided Number
Decahedron with the
numbers 1-10



A Set of Alphabet Cards A-Z

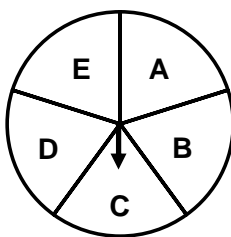


A Spinner



A Bag of 8 Different
Marbles

A 12-sided Number
Dodecahedron with the
numbers 1-12



A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use them to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

Pair 1: A bag of 8 marbles and the 6-sided number cube (48 assignments)

Pair 2: The coin and Set of Alphabet Cards (52 assignments)

Pair 3: The spinner of colors and the 12-sided number dodecahedron (48 assignments)

Pair 4: The spinner with letters and the 10-sided number decahedron (50 assignments)

2. Choose one of the pairs of items above and simulate the event for 50 trials. Create a frequency table to record your results.

Answers may vary experiment to experiment.

3. Create a table in **The Teacher Helper** document under the tab titled **The Choir Helper** to organize the results.

4. Use the spreadsheet to predict the results if the event had been simulated for 100 trials? 250 trials? Make a separate column for each and use formulas to make predictions.

Answers may vary, but formulas should include that the results in #3 are multiplied by a scale factor of 2 for 100 trials and a scale factor of 5 for 250 trials.

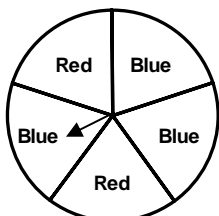
5. Print the document.

Simulation – (Possible Answers)

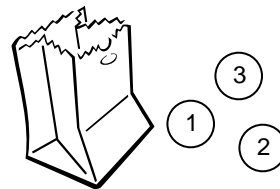
Use the following items to simulate an experiment.



A Coin



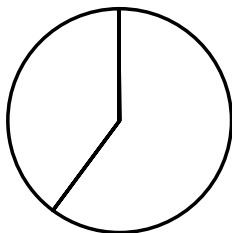
A Spinner



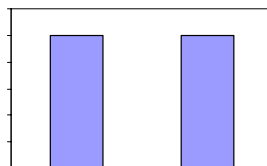
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represents the results of the experiment? Justify your reasoning.

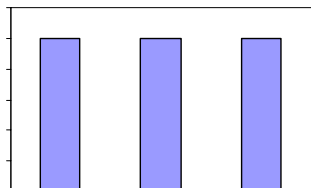
A.



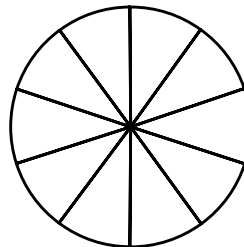
C.



B.



D.

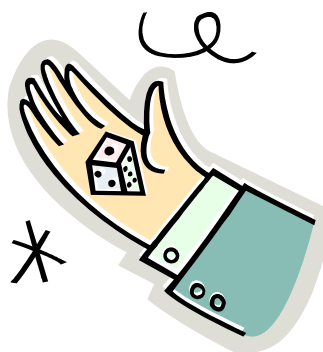
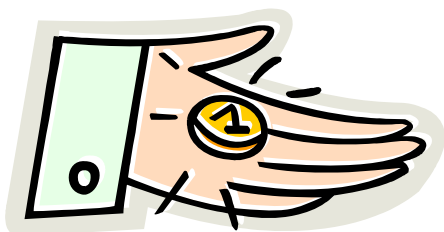


Answer: The graph in A best represents the experiment. In the experiment, the spinner has more blue than red. A circle graph representing the results of blue to red would show a larger section for blue.

Teacher Helper

Mrs. Alexander assigns the job of Teacher Helper in her class by flipping a coin and rolling a 10-sided number decahedron. Each student in her class is assigned a combination of a head or tail and a number from the decahedron. Students in the same class do not share the same combination.

- ❖ If all the possible combinations are assigned, how many students are in Mrs. Alexander's class?
- ❖ What are the possible combinations?



The Helper Dilemma

1. Use a coin and a 10-sided number decahedron to simulate the experiment 40 times. Record your results in the frequency table.

Combination	Tally	Frequency
Head, 1		
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2. Transfer your information into the **The Teacher Helper** document. Follow the instructions in the orange boxes numbered 1-6.
3. Create a graph to represent the Theoretical Probability in Column B.
 - Highlight the Combinations (i.e. Head, 1) in Column A along with the data in the green cells in Column B.
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(continue: The Helper Dilemma)

4. Create a graph to represent the Experimental Probability in Column I (include the Combinations such as Head, 1). Follow the same instructions as #3 except highlight the information in Columns H and I and use the title "Experimental Probability." Drag the chart next to the Theoretical Probability Chart, the first chart.
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6. How were the experimental and theoretical probabilities the same? Explain.
7. How were the experimental and theoretical probabilities different? Explain.
8. If the fractions were changed to percents, what would you expect the percents to total and why?
9. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

10. What is the sample space for this class?
11. What can you conclude about this particular class? Explain.

(continue: The Helper Dilemma)

12. Which gender is most likely to be the helper? Explain.

13. What is the probability of a girl being the helper? Explain.

14. What is the probability of a boy being the helper? Explain.

15. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

The Choir Helper

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code," Mr. Roberts cannot use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

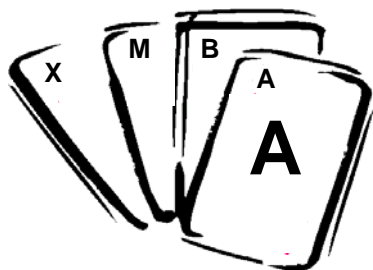


A Coin

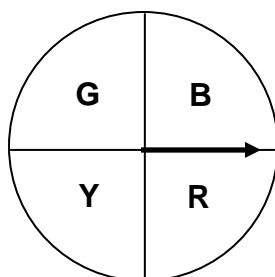


A Six-Sided Number Cube

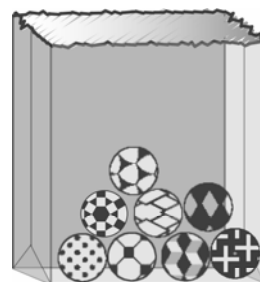
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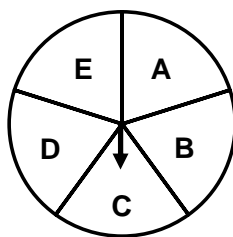


A Spinner



A Bag of 8 Different
Marbles

A 12-sided Number
Dodecahedron with the
numbers 1-12



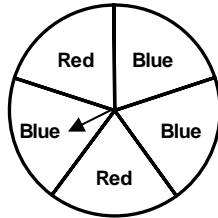
A Spinner

Simulation

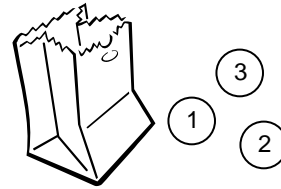
Use the following items to simulate an experiment.



A Coin



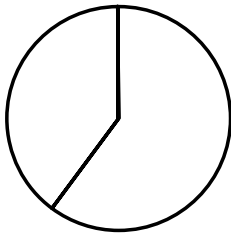
A Spinner



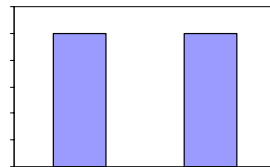
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represents the results of the experiment? Justify your reasoning.

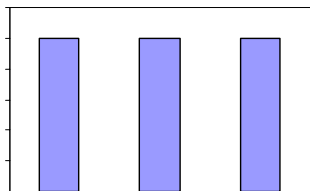
A.



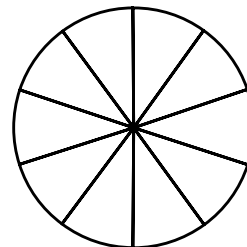
C.



B.



D.



- Corbyn has a standard code of dress at his school. He can wear a white or green shirt with navy or khaki pants. He had 3 white shirts and 2 green shirts in his shirt drawer and 1 pair of navy pants and 3 pairs khaki pants in his pants drawer. What is the probability that Corbyn will reach in both drawers, without looking, and get a white shirt and navy pants?
 - $\frac{17}{20}$
 - $\frac{4}{9}$
 - $\frac{3}{25}$
 - $\frac{3}{20}$

- A 6-sided number cube, a spinner divided into 3 equal parts labeled A, A, B, and a coin are being used for an experiment. Ozzie calculated the theoretical probability of an event where the number cube was rolled, coin tossed, and spinner spun. His calculation was $\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{18} = \frac{1}{9}$. For which of the following events did Ozzie calculate the probability?
 - P(even number, head, B)
 - P(1 or 2, head, A)
 - P(prime number, tail, A)
 - P(odd number, tail, A)

3. The letters of the word WINNER are cut apart and placed in a bag. A letter was drawn from the bag and a coin tossed at the same time. Results were recorded and the letter was placed back into the bag. Which of the following could NOT be used to represent the experimental data?
- A Venn diagram
 - B Bar graph
 - C Circle graph
 - D Line Plot
4. A container of markers containing 3 red, 1 yellow, 2 green and 4 blue are placed at the map center in social studies. The rule is you can only use one marker at a time so that everyone will have a marker to use. What is the probability of reaching into the container without looking for each use and getting a red marker, a blue marker and then a yellow marker?
- A $\frac{3}{250}$
 - B $\frac{12}{30}$
 - C $\frac{12}{100}$
 - D $\frac{8}{10}$